The square root of 2 ain't rational

A Casual Talk By

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Some centuries B.C.

A simple assumption

$$\frac{a}{b} = \sqrt{2}$$
$$\left(\frac{a}{b}\right)^2 = 2$$
$$\frac{a^2}{b^2} = 2$$
$$a^2 = 2b^2$$
$$(2k)^2 = 2b^2$$
$$4k^2 = 2b^2$$
$$2k^2 = b^2$$
$$\frac{a}{b} \neq \sqrt{2}$$

Its consequences

- So what?

$\frac{a}{b} = \sqrt{2}$

$(\frac{a}{b})^2 = 2$

$\frac{a^2}{b^2} = 2$

- $a^2 = 2b^2$

$(2k)^2 = 2b^2$

$4k^2 = 2b^2$

$2k^2 = b^2$

$\frac{a}{b} \neq \sqrt{2}$

The problem

- And but so we said a and b have no common factor.

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$$\frac{a^2}{b^2} = 2$$
$$a^2 = 2b^2$$
$$(2k)^2 = 2b^2$$
$$4k^2 = 2b^2$$
$$2k^2 = b^2$$
$$\frac{a}{b} \neq \sqrt{2}$$

All fractions are reducible

- Suppose $\frac{c}{d}$ is a rational number. If c and d have no common factor, then $a = b$ and $b = d$. If they have a common factor, divide both by their greatest common divisor. The result is $\frac{a}{b}$, with no common factor.

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An even square has an even root

- An even number, by definition, is expressible in the form $2k$, where k is any integer. On the other hand, an odd number is expressible by

$$2k+1$$

Thus the square of an odd number is

$$(2k+1)^2$$

i.e.

$$4k^2+4k+1$$

i.e.

$$2\times 2(k^2+k)+1$$

which is of the form $2k+1$ with $2(k^2+k)$ as k. Hence, an odd number produces an odd square, and thus if a square is even its root is even too.

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- *You might have noticed that this page is slightly scaled to accommodate its content to the slide's declared* `vsize` *parameter. Actually, it is scaled because I stretch this paragraph so as to have too much content. Which is kind of paradoxical. Or just opportunistic.*